

M.Sc. (Previous) Mathematics DEGREE EXAMINATION, MAY 2007**Paper IV - THEORY OF ORDINARY DIFFERENTIAL EQUATIONS**

Time : Three hours

Maximum : 100 marks

Answer any FIVE questions.
All questions carry equal marks.

1. (a) Prove that there exist n linearly independent solutions of

$$Ly = y^{(n)} + a_1(x)y^{(n-1)} + \dots + a_n(x)y = 0.$$

Where $a_i(x), i = 1, 2, \dots, n$ are continuous function on I ?

- (b) Find two linearly independent solutions of the equations

$$(3x-1)y'' + (9x-3)y' - 9y = 0 \text{ for } x > 1/3.$$

2. (a) One solution of $x^2y'' - xy' + y = 0, (x > 0)$ is $\phi(x) = x$.

Find the solution ψ of the equation

$$x^2y'' - xy' + y = x^2.$$

Satisfying $\psi(1) = 0, \psi'(1) = 0$

- (b) Find two linearly independent power series solution of the equation $y'' + 3x^2y' - xy = 0$.

3. (a) Show that the solution ϕ of $y' = y^2$ which passess through the points (x_0, y_0) is given by

$$\phi(x) = \frac{y_0}{1 - y_0(x - x_0)}.$$

- (b) Find an integrating factor for the equation $\cos x \cos y dx - 2 \sin x \sin y dy = 0$ and solve it.

4. State and prove the existence of solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$ on I .

5. (a) Suppose that f is a continuous function on an interval $|x - x_0| \leq a$. show that ϕ is a solution of initial value problem.

$$y' = f(x), y(x_0) = \alpha, y'(x_0) = \beta$$

if and only if it is a solution of integral equation.

$$\phi(x) = \alpha + \beta(x - x_0) + \int_{x_0}^x (x-t)f(t) dt.$$

- (b) Find a solution ϕ of $y'' = -1/2y^2$ satisfying $\phi(0) = 0, \phi'(0) = -1$.

6. (a) Suppose f is a vector valued function defined for (x, y) on a set S of the form

$$|x - x_0| \leq a, |y - y_0| \leq b, (a, b > 0) \text{ or of the forms}$$

$$|x - x_0| \leq a, |y| < \infty (a > 0). \text{ If } \partial f / \partial y_k^{(k=1, 2, \dots, n)}$$

exists, is continuous on S , and there is a constant $K > 0$ such that $\left| \frac{\partial f}{\partial y_k}(x, y) \right| \leq K (K = 1, 2, \dots, n)$ for all (x, y) in S . Prove that f satisfies a Lipschitz condition on S with Lipschitz constant K .

(b) Find a solution ϕ of the system

$$y_1' = y_2$$

$$y_2' = 6y_1 + y_2, \text{ satisfying } \phi(0) = (1, -1).$$

7. (a) Compute the Green's function for the differential equation $y'' - 5y' + 6y = 0$.
- (b) Find the general solution of $y'' - 3y' + 2y = x$ ($-\infty < x < \infty$) by computing the particular solution using Green's theorem.
8. (a) Study the solutions of the Riccatic equation $Z' + Z - e^x Z^2 - e^{-x} = 0$.
- (b) Find four distinct solutions for the differential equation $Z' + Z^2 - 1 = 0$.
9. (a) Show that the equation $x'' + a(t)x' + b(t)x = 0$
Can be transformed into self adjoint equation by multiplying the equation by integrating factor.
- (b) Prove that the Euler's equation $x'' + \frac{k}{t^2}x = 0$
(i) is oscillatory if $K > 1/4$ and
(ii) is non-oscillatory if $K \leq 1/4$
- 10 (a) State and prove Sturm's comparison theorem.
- (b) Let $r(t)$ be a positive continuous function and let m be a real number. Show that the equation
 $x'' + (m^2 + r(t))x = 0, t \geq 0$ is oscillatory.